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Design and Simulation MUSIC Algorithm for DOA Estimation in Comparison to Bartlett and Capon Algorithms for Smart Antenna

Mohamed A Alkelsh[1] , Mohammed M Alshibany[2], and Walid K A Hasan[3]
[1] Electrical and Computer Engineering, University of Gharyan, Gharyan, Libya
[2] Electrical and Computer Engineering, University of Gharyan, Gharyan, Libya
[3] National Authority for Scientific Research , Tripoli, Libya
W.hassan@nu.edu.ly

Abstract. This paper describes three schemes to implement Direction of Arrival Estimation (DOA) that is used and implemented in real Antenna system. The variation between those algorithms is important in means of accuracy and complexity of implementation such mathematical models in actual system of Antenna. The use of mathematical analysis is very helpful to understand the process of DOA estimation and each part of the algorithms in the study, so that can be used further to implement algorithms and work on MATLAB to investigate and evaluate the performance of each one in relative with the others and how they act when they put in same conditions with different number of samples and using different factors. The used algorithms for the study are the conventional algorithms Bartlett & Capon, in contrast with more advanced algorithm MUSIC Multiple-Signal-Classifications, which is introduced as the main focus in the study. Simulation of DOA is made using MATLAB and the mathematical implementation of the three algorithms. The results of comparing the algorithms shows that varying number of samples taken has a crucial role on determining Direction of Arrival or the incident angle of the input stray-signal at the arrays of the smart antenna which senses the direction based on the algorithm operated in the smart antenna system, the use of the three algorithms in different number of samples had shown a performance change of accuracy of estimation in different way for an algorithm in comparison to another algorithm in many circumstances, the more prevailing fact is that the more number of samples the more accuracy of estimation, but in case of less samples the Bartlett algorithm showed better performance in compare to other algorithms, as for Capon algorithm it suffers accuracy at low number of samples and need high number of samples to act more accurately, as for MUSIC algorithm it shows the best overall performance as adapting fast to the angle of estimation with high accuracy in response to an efficient number of samples.
Keywords: Smart antenna, DOA, MUSIC algorithm.

1 Introduction

Currently, the demand for mobile technology services is increasing [1][2], and in order to meet this demand and attract the greatest number of users, new schemes and technologies have emerged in the industry to overcome requirements and bring solutions to these demands, with smart antennas technology being one of these solutions [3][4].

The physical aspect of smart antennas consists of radiating elements in the form of arrays that distribute the receive operation over all elements, hence sophisticating the receiving operation. The array elements are also significant in generating beams with dynamic steering capabilities. [5][6].
Through spatial processing, Smart Antenna technology pushes the physical antenna utilization to its limits, allowing antenna reception of signals from any direction with excellent spectrum efficiency, which is known as space diversity [7][8].

Smart antennas are smart because of the intelligence of the digital signal processing (DSP) implanted in the antenna; the DSP, which contains algorithms, governs the smart antenna's behavior. [9]. There is a DSP unit called direction of arrivals (DOA unit) that acts on algorithms integrated into the antenna's operating system to detect the direction of the signals. Because this unit performs the majority of the smart antenna's functions, the smart antenna's performance is heavily reliant on DOA algorithms. [10] [11].

The smart antennas are now covering all communication fields and playing an important part in improving the quality of connection in all criteria. The smart thing in smart antennas is the Digital Signal Processing unit, which contains the DOA estimation unit that is able to take responsibility on successful transmitting or receiving, where estimation errors can be avoided using high resolution algorithms like the MUSIC algorithm that proposes [11] [12]. However, the objectives of this paper are: Simulating the DOA estimation depending on MUSIC algorithm, Simulating Bartlett and Capon spectrums using the same coding structure. Comparing spectrum of MUSIC algorithm with the spectrum of other two algorithms.

1- Accuracy evaluating for the algorithms in different conditions first with varying number of samples and second with varying signal to noise ratio SNR.
2- Calculating the estimation errors from the collected results for all algorithms and presenting it as curves to show the overall performance of algorithms.

The rest of the paper will be organized as follows: Section 2 Simulation steps of MUSIC algorithm will be introduced in addition to coding less grade algorithms (Conventional Algorithms) using the same structure, this is accomplished using MATLAB. Section 3 shows the simulation results and accuracy evaluation for the algorithms. Section 4 contains the overall conclusion and the future work of this paper.

2 Simulation and Accuracy Evaluation

2.1 Validity of Simulation

The simulation is based on the analytical and mathematical representation of MUSIC algorithm, it’s an implementation of the official algorithm and the calculations done in MATLAB for simulation that shows the mathematical process of the algorithm needed to estimate the direction of the incoming signals.

2.2 Simulation Description & Steps

Direction of arrivals estimation unit simulation on smart antenna at base station receiver (Uplink) of mobile communication is based for estimation parameters of Azimuth angles and Uniform Linear Array ULA antenna type.

In the simulation section, it presents the calculations of DOA parameters in MATLAB in term of steps, as follows:

Step 1: Incoming signal

The incoming signal is a Far-field narrow band signal that always propagates in speed of light and occupies all the spectrum of the carrier signal thus:

$$\frac{f_c}{f_{Propagation}} = 1 \qquad (1)$$

So, in this case the operating frequency is (2GHz) for mobile communications and has no effect on the DOA estimation process.

Once the signal is received, the signal is down-converted and digitally demodulated and become a baseband signal, by using a binary PSK symbols it can be generated in baseband.

Step 2: Environment Impairments (Noise**)**

It is assumed that the noise components are to be Additive White Gaussian Noise (AWGN). This models (forms) the background noise of the communication channel. This uncorrelated White Gaussian Noise, it can be generated in this step using MATLAB.

Step 3: Array Response

The sensor array response for a coming signal from a specific direction can be represented in this equation:

$$a(\emptyset_i) = [1, e^{-jw\tau_2(\emptyset_i)}, \dots\dots\dots\ e^{-jw\tau_m(\emptyset_i)}]^T \qquad (2)$$

$$\text{Where, } \tau = \frac{d \sin\theta}{c} \qquad (3)$$

an implementation for the above equation can be done in MATLAB which shows Sensor Array Response.

Step 4: Array Input signal

The overall input signals to the array can be represented in data matrix form:

$$x(t) = \sum_{i=1}^{M} a(\emptyset_{i-1})S(t) + n_i(t) \qquad (4)$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_m(t) \end{bmatrix} = S(t) \begin{bmatrix} 1 \\ e^{-j\emptyset} \\ e^{-j2\emptyset} \\ \vdots \\ e^{-j(m-1)\emptyset} \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_m(t) \end{bmatrix}$$

Step 5: Spatial covariance matrix

Because the parameters of the DOA estimation are spatial in real, the spatial covariance matrix is needed to be generated and number of samples (K) are taken in discrete periods:

Step 6: Compute Eigen-decomposition of covariance matrix

This transforming the covariance matrix into Eigen values and Eigen vectors.

Step 7: Sort for largest Eigen values and Eigen vectors (Descending)

This step is rearrangement for the Eigen data and can be done in MATLAB.

Step 8: Separate Eigenvectors into (signal and noise) Subspaces

In this step the Eigen vectors are separated.

Step 9: Spatial Spectrum (DOA)

Now it is needed to create a space to put all results in, as results of the estimation process, the Spatial spectrum presents the distribution of signals in every direction in the space, the spatial spectrum of MUSIC algorithm is represented by this equation:

$$P_{MUSIC}(\varphi) = \frac{a(\varphi)^H a(\varphi)}{a(\varphi)^H E_n E_n^H a(\varphi)} \quad (5)$$

In this case the steps should be taken as follows:

- Define angles for the MUSIC "spectrum".
- Compute steering vectors related to the spectrum angles.
- Generate the MUSIC "spectrum".

Step 10: Coding Conventional algorithms

To show the ability of our algorithm in this project which is the MUSIC algorithm, a compare with less grade algorithms (Conventional) should be made, so by using the same first five steps of the last explanation of MUSIC algorithm, and continuing with the following explanation, the conventional algorithms can be implemented:

- ***Bartlett Algorithm:***

$$P_{Bar}(\varphi) = a(\varphi)^H \, R_{xx} \, a(\varphi) \quad (6)$$

This can be implemented as follows:

- Define angles for Bartlett "spectrum".
- Compute steering vectors related to spectrum angles.
- Generate Bartlett "spectrum".

- ***Capon Algorithm:***

$$P_{Capon} = \frac{1}{a^H(\varphi) R_{xx}^{-1} a(\varphi)} \quad (7)$$

This can be implemented as follows:

- Define angles for Capon "spectrum"
- Compute steering vectors related to spectrum angles
- Generate Capon's Minimum Variance "spectrum".

3 Simulation Results and Discussion

In mobile communication system, smart antennas at base stations (uplink) has specifications for the DOA estimation unit, in this simulation the used parameters and standards are given in table (1).

Table (1) simulation parameters in DOA unit

	Standards	value
1	Number of array elements (M) for azimuth	4 elements
2	Spacing between elements is $(d = \lambda/2)$	$(D) = (d/\lambda = 1/2)$
3	Number of snapshots (samples) K	$K = 1024$
4	Power of incoming signals P	$P = 1$
5	power spectral density (σ^2) of noise	$(N_0/2)$ where $(N_0 = 1)$

6	Angles of Arrivals (AOA)	(-30 0 30)

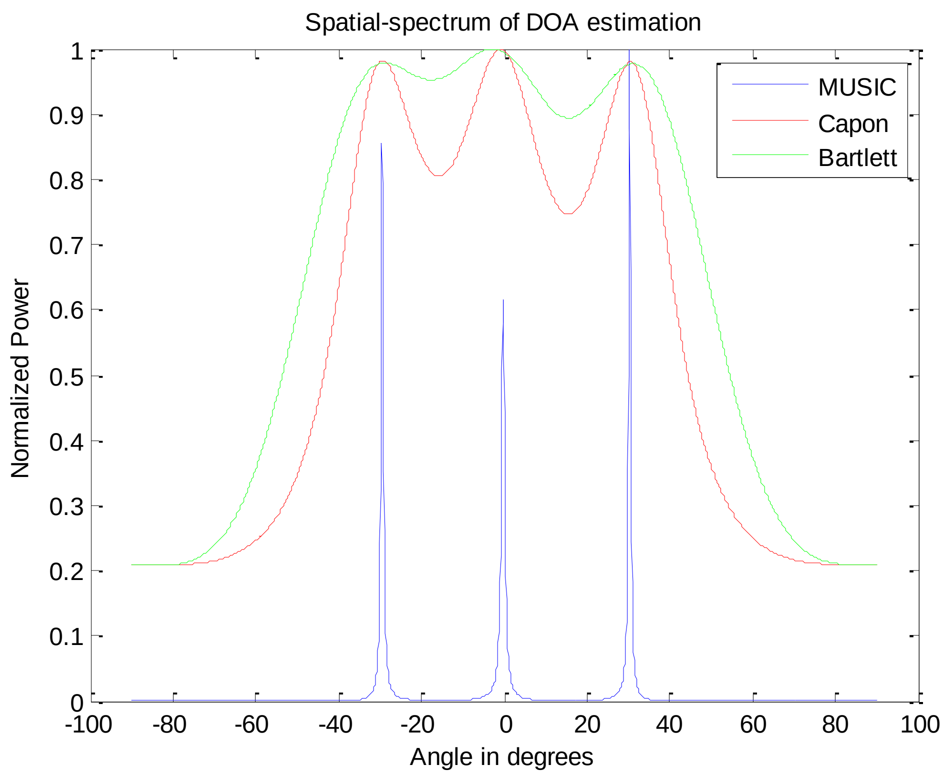

Fig. 1. DOA Simulation results of (MUSIC, Capon, Bartlett) algorithms

MUSIC algorithm in fig. (1) has the best performance for the three directions (-30, 0 , 30) with sharp peaks that results in accurate estimation. As for, Capon algorithm has the second-best performance with thick peaks that results in degraded accurate estimation. Whereas, Bartlett algorithm has the lowest performance with less variance between peaks (Undistinguishable) which gives inaccurate estimation.

4.1 Accuracy evaluation for the algorithms

For this procedure, the accuracy of algorithms will be evaluated in different conditions of (Snapshots and Signal to noise ratios), and for three angles (-30, 0, 30), and to handle these angles, the number of array elements are doubled to 8.

Varying Number of Samples (Snapshots):

Here the number of samples will be varied and the SNR will be stained at 13 dB which is the standard ratio of last simulations, the number of snapshots here are chosen as follows:

- **Minimum Snapshots (1 sample):**

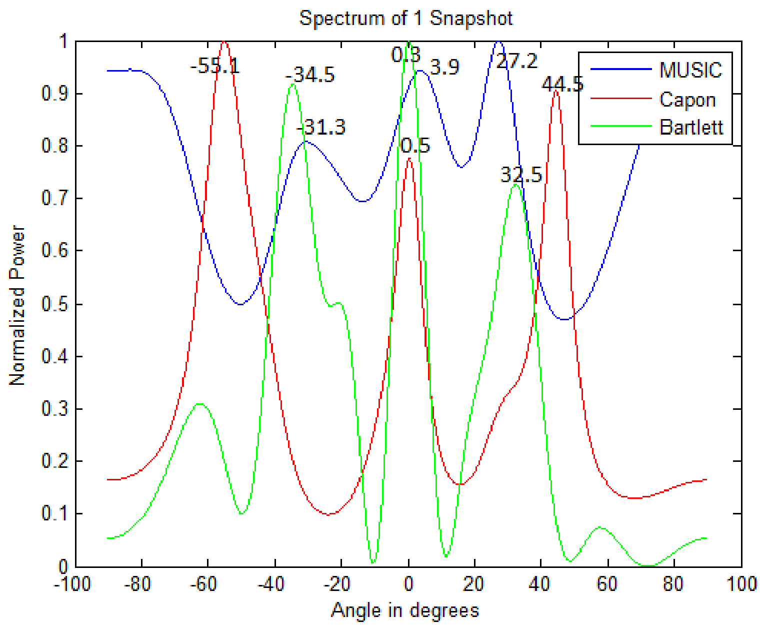

Fig. 2. DOA estimation of 1 sample

- **Low Snapshots (8 samples):**

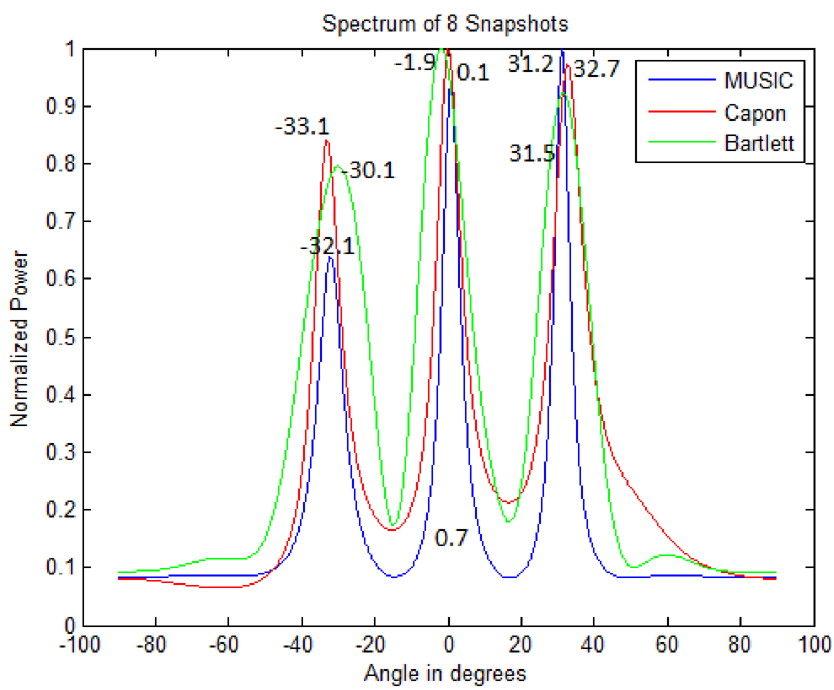

Fig. 3. DOA estimation of 8 samples

- **Medium Snapshots (16 sample):**

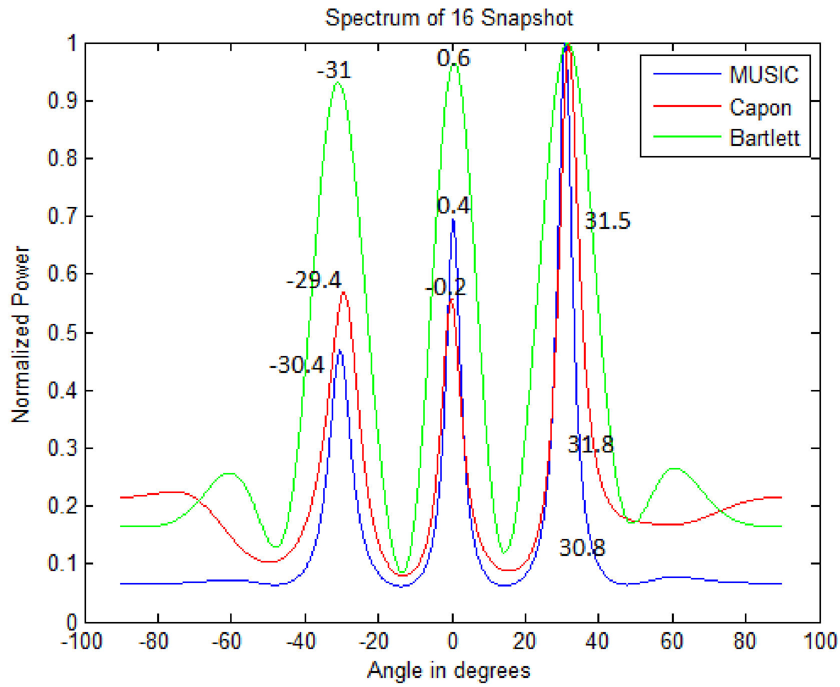

Fig. 4. DOA estimation of 16 sample

- **High Snapshots (64 sample):**

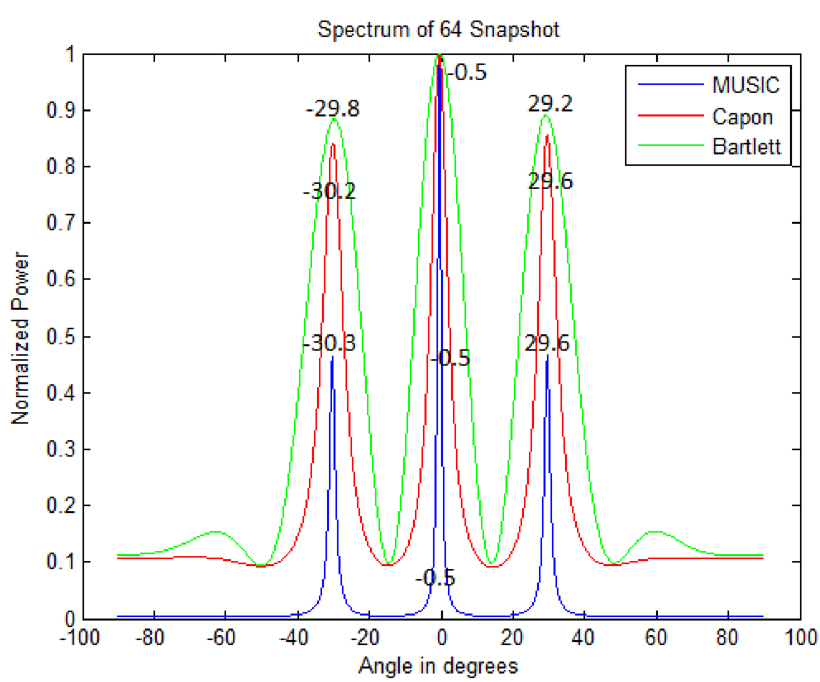

Fig. 7. DOA estimation of 64 sample

Discussion of samples-varying results:

- For minimum snapshots (1sample) the best performance was for Bartlett algorithm then MUSIC then Capon.
- For low snapshots (8 sample) the best performance was for Bartlett algorithm then MUSIC then Capon.
- For medium snapshots (16 sample) the best performance was for MUSIC algorithm then Capon then Bartlett.
- For high snapshots (64 sample) the best performance was for Capon and MUSIC then for Bartlett.

4.2 Varying SNR (signal to noise ratio):

In the last simulations the calculated SNR was 13dB which is the standard SNR for practical purposes, here the SNR will be varied and the number of snapshots are chosen of (32 sample) with same number of elements of the last procedure 8 elements.

$$SNR = 20 * \log(\frac{P}{N_0/2})$$

$$Low_{SNR} = 20 * \log(\frac{1}{2/2}) = 20 * \log(1) = 0\ dB$$

$$Medium_{SNR} = 20 * \log(\frac{1}{1/2}) = 20 * \log(2)\ = 13\ dB$$

$$High_{SNR} = 20 * \log(\frac{2}{1/2}) = 20 * \log(4) = 27\ dB$$

- **Low SNR (0 dB)**

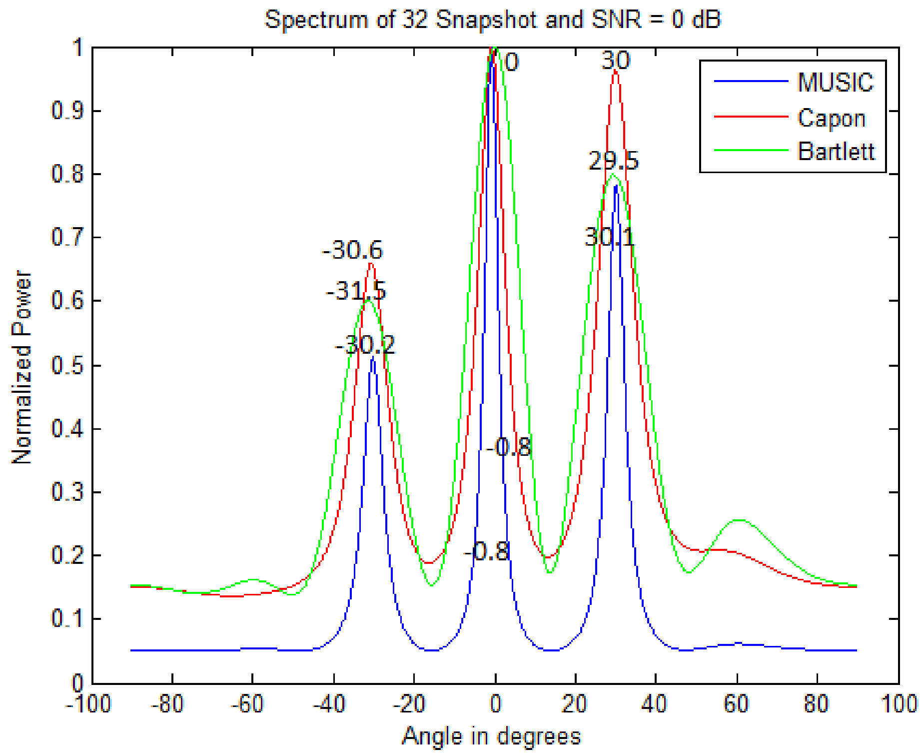

Fig. 8. DOA estimation with SNR = 0 dB

- **Medium SNR (13 dB)**

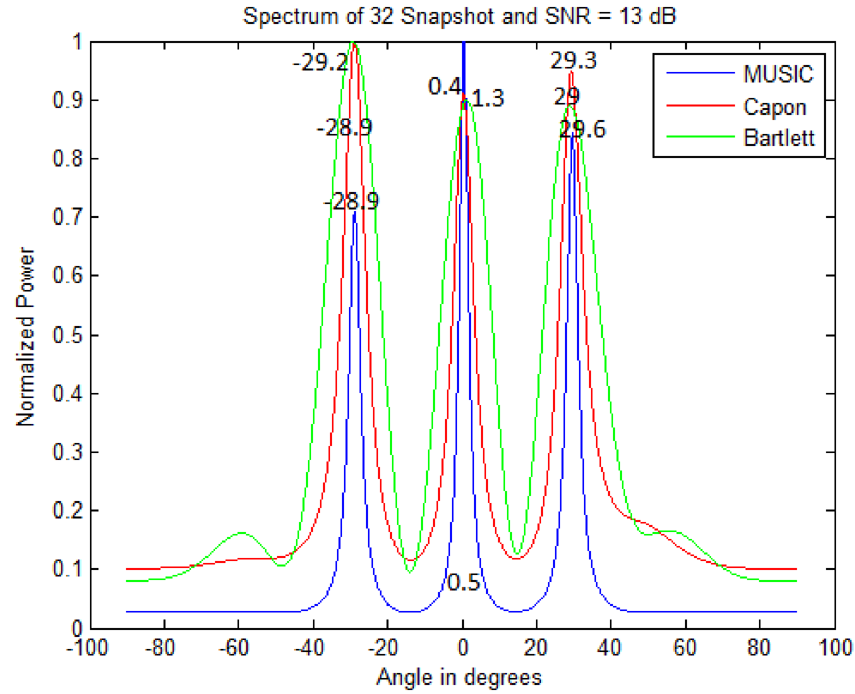

Fig. 9. DOA estimation with SNR = 13 dB

- **High SNR (27 dB)**

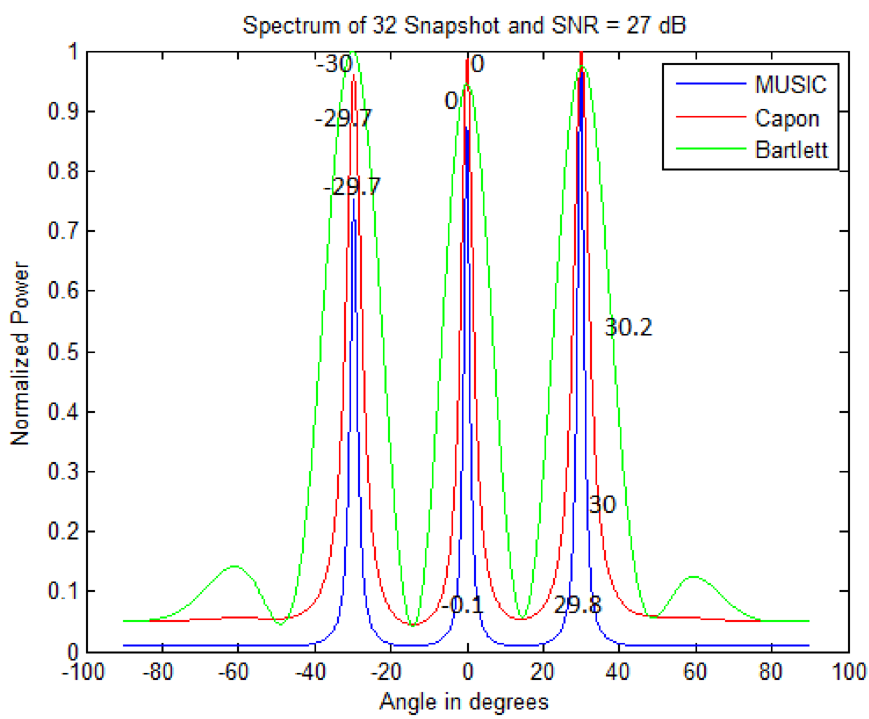

Fig. 10. DOA estimation with SNR = 27 dB

Discussion of SNR-varying results:

- For low SNR (0 dB) in figure(13) the best performance in this noisy condition is for MUSIC algorithm with error of (1.1°), coming after that; Capon algorithm with error of (1.4°), and the less performance is for Bartlett algorithm with error of (2.0°).
- For medium SNR (13 dB) in figure(14) the best performance is for MUSIC algorithm with error of (2.0°), coming after that; Capon algorithm with error of (2.2°), and the less performance is for Bartlett algorithm with error of (3.1°).
- For high SNR (27 dB) in figure(15) the best performance is for Bartlett algorithm with error of (0.2°), coming after that; Capon algorithm with error of (0.3°), and the less performance is for MUSIC algorithm with error of (0.6°).

Estimation errors of the algorithms:

From the last evaluation for the performance of the three algorithms and as a result, the estimations are collected and the range of error for every algorithm is calculated and put in one table, this table showing the accuracy of estimations in different cases of snapshots:

Table (2) Data of accuracy Results

No. of Snapshots	Bartlett Algorithm (-30 , 0 , 30)			Capon Algorithm (-30 , 0 , 30)			MUSIC Algorithm (-30 , 0 , 30)			Total Errors (degree)		
1	-34.5	0.3	32.5	-55.1	0.5	44.5	-31.3	3.9	27.2	7.3	40.1	8
8	-30.1	-1.9	31.5	-33.1	0.1	32.7	-32.1	0.7	31.2	3.5	5.9	4
16	-31	0.6	31.5	-29.4	-0.2	31.8	-30.4	0.4	30.8	3.1	2.6	1.6
32	-29.2	1.3	29	-28.9	0.4	29.3	-28.9	0.5	29.6	3.1	2.2	2
64	-29.8	-0.5	29.2	-30.2	-0.5	29.6	-30.3	-0.5	29.6	1.5	1.1	1.2

- The above table are summarized and represented as curves of estimation errors:

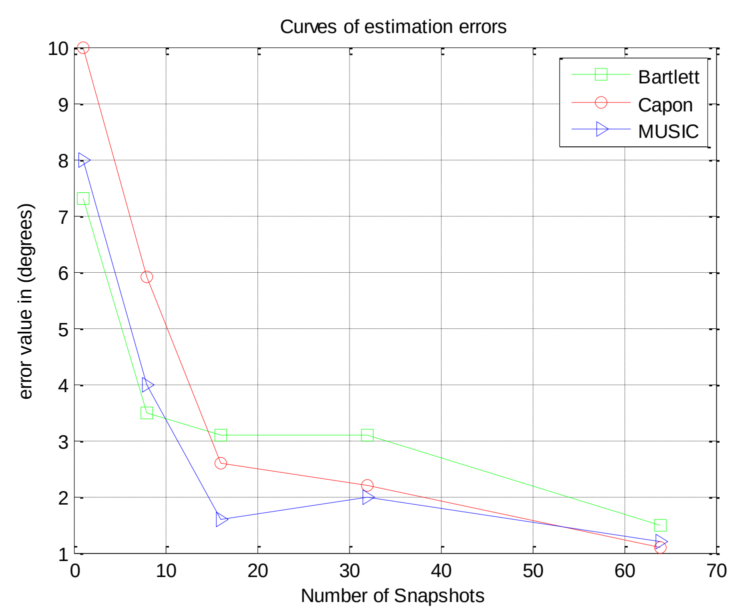

Fig. 11. Curves of estimation errors

Discussion of curves of estimation errors:

From last fig. (11) some notes can be obtained:

- Bartlett algorithm has the best performance in case of minimum samples (snapshots) at about less than 10 snapshots with less estimation errors, while a degradation in performance is obvious in other cases for more snapshots, and this makes the overall performance of Bartlett algorithm is weak and at the same time is a solution for low snapshot conditions.
- Capon algorithm has the greatest error at the beginning for case of less than 10 samples but after that there is a great drop of the error value until the end of curve, and that's make Capon algorithm is the best choice for high snapshots conditions, in other words Capon algorithm needs high processing to give better results.
- MUSIC algorithm has a moderate performance at the beginning for low snapshot conditions, but from 10 snapshot to the rest, the MUSIC algorithm has the best performance overall, for that; MUSIC algorithm can be considered as the algorithm that has the best performance.
- **To summarize**, the more prevailing fact is that the more number of samples the more accuracy of estimation, but in case of less samples the Bartlett algorithm showed better performance in compare to other algorithms, as for Capon algorithm it suffers accuracy at low number of samples and need high number of samples to act more accurately, as for MUSIC algorithm it shows the best overall performance as adapting fast to the angle of estimation with high accuracy in response to an efficient number of samples.

Here are some concluded points that can be summarized:

- The kind of algorithm to be used for DOA estimation must be selected carefully depending on the required quality and the operation conditions.
- The MUSIC algorithm has proved its performance in all conditions compared with other algorithms, so in these circumstances the MUSIC algorithm can be selected as a solution for high resolution (Quality) requirements.
- The control ability of number of snapshots taken can overcome the weakness in some algorithms.

4 Conclusion and Future Work

MUSIC algorithm can be considered as an algorithm with high performance and stability. The more advantageous thing is, this algorithm can handle sources more than other algorithms while maintaining high accuracy for most sources, and this increases the spectral efficiency of smart antennas that use this algorithm. For the MUSIC algorithm it is recommended to study further the schemes of how to improve estimation and implement the improved Root-MUSIC algorithm using MATLAB, and to compare with other high-resolution algorithms such as ESPRIT algorithm and WSF algorithm to evaluate its performance.

Conflict of Interest

This is to certify that all authors have seen and approved the manuscript being submitted and they declare no competing interest.

References

1. Hasan, W. K., & Zergalin, O. A. CELLULAR MOBILE RADIO TELECOMMUNICATION NETWORK PLANNING TECHNOLOGY - Journal of Engineering Research and Applied Science. 2018
2. Zergalin, O., & Hasan, W. K. (2018). Post Study Design of Wireless Network for the Water Dilemma in the city of Rujban. The International Journal of Engineering and Information Technology (IJEIT), 4(2).
3. Satrusallya, S. (2021). Evaluation of Beam Forming Capability of Linear Antenna Array for Smart Antenna System. *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, *12*(9), 637-642.4. Dakulagi, V., & Bakhar, M. (2020). Advances in smart antenna systems for wireless communication. Wireless Personal Communications, 110(2), 931-957.
5. Boustani, B., Baghdad, A., Sahel, A., & Badri, A. (2019). Performance analysis of direction of arrival algorithms for smart antenna. International Journal of Electrical and Computer Engineering, 9(6), 4873.
6. Shashank, B., Raghavender, G., & Choubey, S. B. ADAPTIVE BEAM FORMING IN SMART ANTENNA FOR MOBILE COMMUNICATION.
7. Oluwole, A. S., & Srivastava, V. M. (2018). Analysis and Synthetic Model of Adaptive Beamforming for Smart Antenna Systems in Wireless Communication. J. Commun., 13(8), 436-442.
8. Shahid, M. U., Rehman, A., Mukhtar, M., & Nauman, M. (2020). Analysis of Fixed Beamforming Algorithms for Smart Antenna Systems. International Journal of Electronics and Communication Engineering, 14(5), 110-116.
9. Oluwole, A. S. (2018). Analysis and design of smart antenna arrays (SAAs) for improved directivity at GHz range for wireless communication systems (Doctoral dissertation).
10. Yuvaraj, R., & YV, A. K. (2022). Performance Analysis of Optimal Analog Beamforming with Modified ESPRIT DOA Estimation Algorithm for MIMO NOMA System Over Rayleigh Fading Channel. ECS Transactions, 107(1), 11361.
11. KHUMANE, D., & JAGADE, D. (2019). Smart Antenna: DOA Estimation and Beamforming using MUSIC & LMS Algorithms.
12. Dakulagi, V., Dakulagi, R., Yeap, K. H., & Nisar, H. (2022). Improved VSS-NLMS Adaptive Beamformer Using Modified Antenna Array. Wireless Personal Communications, 1-12.