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# Comparative Numerical Simulation of Laminar Flow Using FVM and Commercial Software

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**Abstract:**
In this paper, the flow behaviour in a pipe with sudden contraction is studied numerically using the finite volume method. A general code with automatic mesh generation is created and validated compared to numerical and experimental results. The effects of the geometry dimensions and Reynolds number on the flow are evaluated for laminar flow modes. Numerical predictions of the flow employing a finite volume computer code were also undertaken. This code was carefully checked and optimized to yield reliable predictions for pipe flows with a sudden contraction in cross-sectional area. Computational results are presented and compared with the available experimental results. The stability of the solution is controlled by the use of the equal order element for the velocity and the pressure. The results are found to be very sensitive to the stabilization parameters. The current results are closer to the experimental results than other numerical results in the references. This investigation is carried out to gain insight into the flow structure near the sudden contraction and understand the increased pressure losses generated in this region.

**Keywords:** lamina flow, pipe flow, finite volume method, fluent.

## 1. Introduction

Fluid flow in ducts and pipes is a commonly encountered phenomenon in engineering practice and, as such, requires special attention. A pressure difference must exist between the inlet and the outlet for a flow to occur. This pressure difference determines the flow rate**.** Therefore, the flow system's designer needs to estimate the pressure difference for the flow rate to be in a position to select a suitable pump for his system. Thus, it is apparent that a poor estimate of the pressure difference will lead to the flow system operating away from its optimum condition, which could be detrimental to both system performance and operating costs**.** In practice, flow systems or pipelines do not consist entirely of straight lengths of pipe where the flow is fully developed. Instead, they consist of entry length, bends, valves, fittings, changes in cross-section such as a contraction from a large pipe bore to a small bore and enlargements. Contractions exist in various processes, including chemical plants (heat exchangers, fired heaters and boilers), where they occur in conjunction with enlargements and at the entrance to the tube bundles attached to the manifolds or headers. Contraction also occurs in conjunction with enlargements because of the use of ferrules for close control of the flow distribution within tube bundles. The differential ferrule is often used to ensure that each tube receives the desired flow, consistent with its heat input and pressure loss characteristics**.**

A schematic diagram of fluid flow through pipe contraction is shown in Figure (1.1). As the fluid flows through the large bore pipe, energy dissipation is due to fluid friction. At some location (2) Figure (1) upstream of the contraction, the fluid separates from the pipe wall and is directed towards the entrance of the small-bore pipe**.**

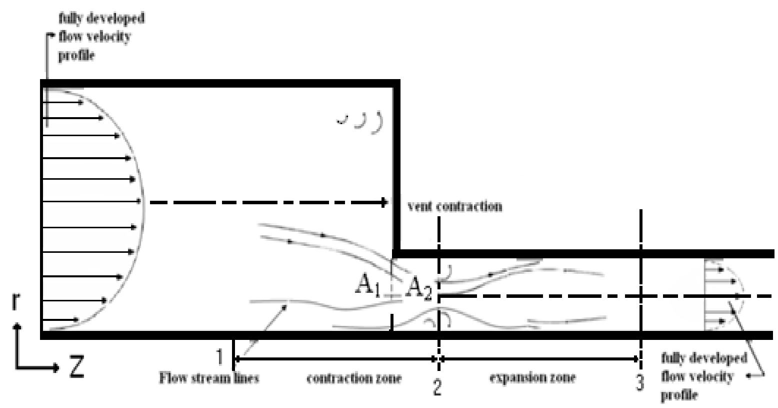


Figure (1) Flow in a Pipe with Sudden Contraction

The streamlines continue inwards towards the pipe center past the contraction (A1/A2) to form a minimum flow area called the vent contract. It is shown as location (2) in Figure (1) [1]. Downstream the vent contract, the fluid expands to re-attach at location (3) to the walls of the small-bore pipe. The pressure loss due to the contraction thus occurs from the point of separation in the large bore pipe to the point of re-attachment in the small-bore pipe.

The contraction process thus consists of a contraction in the flow area from the large pipe bore to the vent contract, followed by an expansion process from the vent contract to the small-bore pipe. The mechanism responsible for the additional pressure loss caused by the change in area is the generation of large-scale turbulence that extracts energy from the mean flow. Accurately predicting contraction pressure loss is essential when it is a significant component of the overall system pressure loss or when it can significantly affect the flow conditions. These situations are most commonly encountered in heat exchangers, particularly fired heaters, boilers, cooling water condensers etc. [2]

## 2. Literature Survey

The current research deals with the velocity estimation of a flow in a pipe with sudden contraction. Numerous experimental and numerical research works on a flow estimation of similar geometries were performed before, and many are going on. A three-step approach is the most accepted approach to laminar flow estimation of various geometries adopted by most researchers in this area. This three-step approach predicts the flow field in the domain of interest, calculating the friction factor and predicting the temperature distribution from the data obtained in the previous steps .

This section serves as a literature review of previous works done by other researchers, which has been used as reference sources, support and background for this research. Many papers and books have been consulted, but most are briefly mentioned, and some are discussed in this study. The papers with more significant contributions to the field are discussed here.

Durst et al. (1985) [3] presented an experimental study for laminar flow in a pipe with sudden contraction for laminar flow regions. A comprehensive study for the Reynolds number range (based on the upstream pipe diameter) of 23 to 1213 for an area ratio of 0.285 using a Dual Beam LDA system operating in the forward scatter mode with signal processing by a frequency tracker or transient recorder .

Mounir Ibrahim and Waqar Hashim (1992) [4] study based on the pattern of this problem and the possible ways to solve it.

The heat transfer results in oscillating flows with a sudden cross-section change are presented in Mounir's work et al. [4]. Oscillating fluid flow with heat transfer between two parallel plates with a sudden change in cross-section was examined computationally. Chen and David (1994) [5] presented numerical and experimental studies of particle deposition in a tube with a conical contraction-laminar flow regime. Particle deposition in a tube with a conical contraction was studied numerically and experimentally . Fossa and Gugliemini (1998) [6] experimentally investigated the void fraction in horizontal pipes with sudden contraction area. The experiments aimed to analyse singularity characteristics' effect on void fraction profiles and phase distribution . Oliveira et al. (2003) [7] presented a study on the effect of contraction ratio in viscoelastic flow through abrupt contractions. A numerical study of the flow through planar sudden contractions was carried out to quantify the effect of contraction ratio upon the flow characteristics (streamlines and size and intensity of recirculation vortices( .Afonso and Pinho (2005) [8] presented a Numerical investigation of the velocity overshoots in the flow of viscoelastic fluids inside a smooth contraction. Rogério and José (2007) [9] presented laminar elliptic flow in the entrance region of tubes. The developing region of an axially symmetric laminar flow from a reservoir to a sharp-edged tube is numerically simulated with a primitive-variables solver of the Nervier-Stokes equations, using a plenum upstream of the tube inlet in order to avoid arbitrary specification of the profile at the inlet. Development region lengths, velocity profiles and head loss are obtained for various Reynolds numbers. Notably, none of the investigations has considered non-axisymmetric pipe as their required geometry. Instead, all the literature discussed so far provided data for the qualitative development of the flow through sudden pipe contraction only. However, the data needs to be more detailed to enable a comprehensive correlation of the flow field. Invariably there are problems with such correlations since all the relevant data, such as inlet flow conditions and non-axisymmetric pipe, are not well known for all the studies. This makes it very difficult to compare the various studies. Therefore, the finite volume analysis has been considered here for two cases, axisymmetric and non-axisymmetric pipe (sudden contraction). The axisymmetric pipe flow is validated using the existing literature mentioned above. Since there is no direct comparison in the literature with the current non-axisymmetric pipe flow, the Fluent software is used to validate this study.

## 3. Conservation laws of fluid motion

The governing equations of heat transfer and fluid flow (i.e. the conservation of mass, momentum and energy equations) are considered along in this paper. These equations are formulated for steady laminar flow; two-dimensional flow configuration and consider the following hypothesis.

- Incompressible and laminar flow

- Newtonian behavior
- Physical properties are constants
- Viscous dissipation in the energy equation is negligible

Under these hypotheses, it is possible to cover a wide range of engineering applications.

1.1 Mass conservation in three dimensions

$$\frac{1}{r}\frac{\partial(rv_r)}{\partial r}+\frac{1}{r}\frac{\partial(v_\theta)}{\partial\theta}+\frac{\partial v_z}{\partial z}=0 \tag{1}$$

1.2 Momentum equation in three dimensions

$$\rho\left(\frac{\partial v_r}{\partial t}+\frac{1}{r}\frac{\partial(rv_r v_r}{\partial r}+\frac{1}{r}\frac{\partial(v_\theta v_r)}{\partial\theta}-\frac{{v_\theta}^2}{r}+\frac{\partial(v_z v_r)}{\partial z}\right)$$
$$=-\frac{\partial p_d}{\partial r}+\left[\frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta r}}{\partial\theta}-\frac{\tau_{\theta\theta}}{r}+\frac{\partial\tau_{zr}}{\partial z}\right] \tag{2}$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+\frac{1}{r}\frac{\partial(rv_r v_\theta}{\partial r}+\frac{1}{r}\frac{\partial(v_\theta v_\theta)}{\partial\theta}+\frac{v_r v_\theta}{r}+\frac{\partial(v_z v_\theta)}{\partial z}\right)$$
$$=-\frac{1}{r}\frac{\partial p_d}{\partial\theta}+\left[\frac{1}{r^2}\frac{\partial(r^2\tau_{r\theta})}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta\theta}}{\partial\theta}+\frac{\partial\tau_{z\theta}}{\partial z}\right] \tag{3}$$

$$\rho\left(\frac{\partial v_z}{\partial t}+\frac{1}{r}\frac{\partial(rv_r v_z}{\partial r}+\frac{1}{r}\frac{\partial(v_\theta v_z)}{\partial\theta}+\frac{\partial(v_z v_z)}{\partial z}\right)$$
$$=-\frac{\partial p_d}{\partial z}+\left[\frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r}+\frac{1}{r}\frac{\partial\tau_{\theta z}}{\partial\theta}+\frac{\partial\tau_{zz}}{\partial z}\right] \tag{4}$$

Where shear stresses are evaluated considering the Newtonian behavior by means of Stokes law:

$$\tau_{rr} = 2\mu\left(\frac{\partial v_r}{\partial r}\right),$$

$$\tau_{r\theta} = \tau_{\theta r}=\mu\left(r\frac{\partial\left(v_\theta/r\right)}{\partial r}+\frac{1}{r}\frac{\partial v_r}{\partial\theta}\right),$$

$$\tau_{\theta\theta} = 2\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial\theta}+\frac{v_r}{r}\right),$$

$$\tau_{\theta z} = \tau_{z\theta}=\mu\left(\frac{\partial v_\theta}{\partial z}+\frac{1}{r}\frac{\partial v_z}{\partial\theta}\right),$$

$$\tau_{zz} = 2\mu\left(\frac{\partial v_z}{\partial z}\right),\ \tau_{zr} = \tau_{rz}=\mu\left(\frac{\partial v_z}{\partial r}+\frac{\partial v_r}{\partial z}\right)$$

1.3 Energy equation in three dimensions

$$\frac{\partial T}{\partial t}+\frac{1}{r}\frac{\partial\left(rv_r T\right)}{\partial r}+\frac{\partial\left(v_z T\right)}{\partial z}=\frac{k}{\rho c_p}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial Z^2}\right) \tag{5}$$

Notice that components of the heat flux vector entering the unit of volume by conduction are evaluated using Fourier's law:

$$\vec{q} = -k\nabla T_m$$

## 4. The results and discussions

The ideal way to validate the single-phase CFD predictions is to compare the predicted velocity fields with the actual velocity fields. For the validation purpose in the present work, the paper of F. Durst & T. Loy (1983) is chosen [2]. For the present numerical work, the flow computations were performed using the FORTRAN POWER STATION 4 to write a Fortran CODE and compare it with the computational fluid dynamics package FLUENT. This specific package uses the finite volume method to discretize the Navier-Stokes equation. Initially, the convergence of the continuous phase for all the flow variables was set at $10^{-5}$ using the second-order scheme for momentum and laminar quantities. Then, to get more precise results, higher-order convergence criteria of $10^{-6}$ are set for all the flow variables.

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**Case 1 axisymmetric pipe**

This study presents numerical results using the Fluent and Fortran code for lamina flow in a pipe with sudden contraction. The axial-velocity contours in an axisymmetric pipe with sudden contraction using Fluent are shown in Figure (2). The obtained results using the Fluent and Fortran code are compared with Durst and Loy [2] experimental data for three different Reynolds numbers Red(371, 704, and 1831), as shown in Figures(3-9). The current study results using the Fortran code are in good agreement with Durst and Loy's experimental data, and it is in better agreement than Fluent results, as illustrated in Figures (3-9).

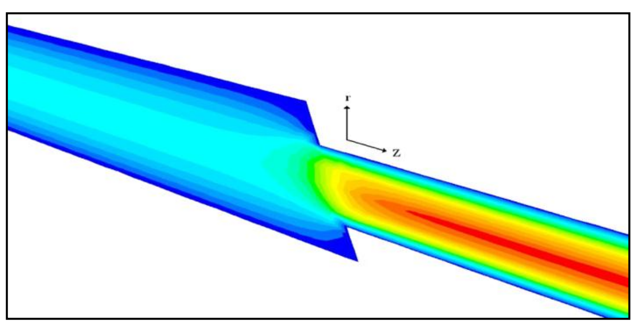


Figure (2) Axisymmetric pipe with contraction ratio = 1.86

The experimental data for Durst and Loy for a pipe Section of Z/d=1.961, Cr=1.86 and Red= 371,704,1831 are compared with the current study's numerical results. This comparison provides insight into the pipe flow with sudden contraction in the cruise section; further information can be gained from both the experimental and the numerical results, as shown in Figures (3) and (4). In addition, Figures (3) and (4) disclose measurements and predictions of distributions of the axial velocity component. As is evident from the Figures, there is a good agreement between the experimental and the obtained numerical results from this study.

**results from this study.**

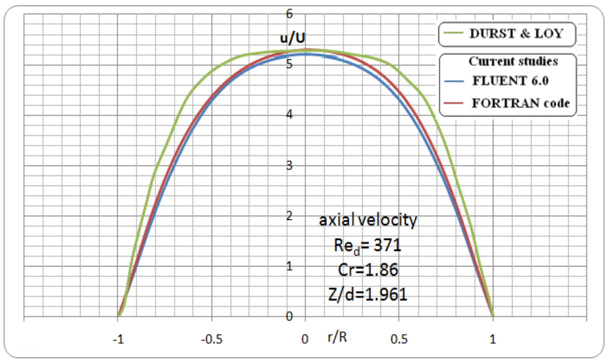


Figure (3) Comparison the axial velocity profile for the FORTRAN code, fluent software & Durst

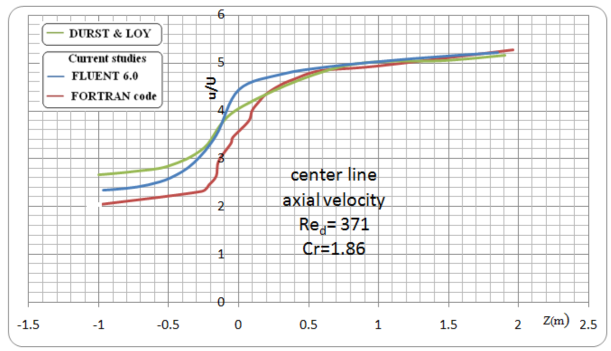


Figure (4) Comparison the centerline axial velocity profile for the FORTRAN code, fluent software & Durst

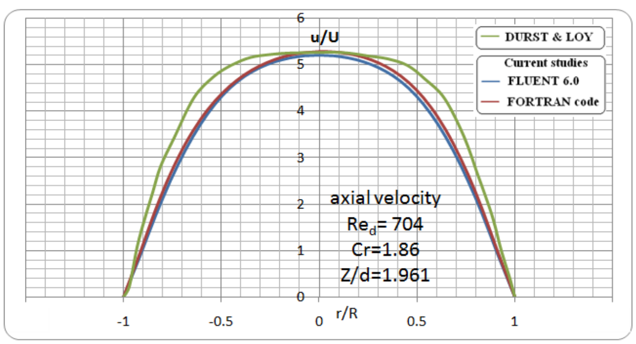


Figure ( 5) Comparison the axial velocity profile for the Fortran code, fluent software & Durst

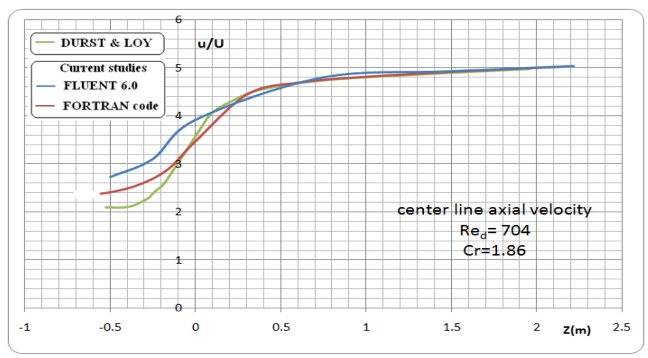


Figure (6) Comparison the centerline axial velocity profile for the FORTRAN code, fluent software & Durst

For the case of ReD =968, using the same geometry, Cr=1.86 at Z/d=1.96, there is a somewhat more significant discrepancy between the experimental and the numerical results of this study. It is worth mentioning that the flow at this Reynolds number ReD =968 in the large pipe produces a flow of Red =1831 in the smaller pipe, depending on the contraction ratio. This flow is close to the critical Reynolds number so that the flow downstream of the plane of contraction is in the region of laminar-turbulent transition. The present numerical codes cannot handle this kind of flow, whereas the experiments are taking the transitional behavior fully into account.

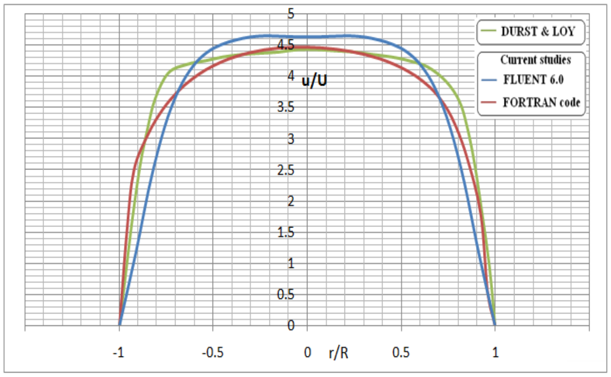


Figure (7) Comparison the axial velocity profile for the FORTRAN code, fluent software & Durst

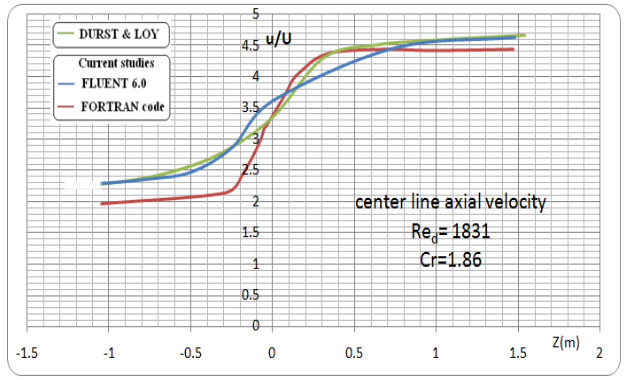


Figure (8) Comparison the centerline axial velocity profile for the FORTRAN code, fluent software & Durst

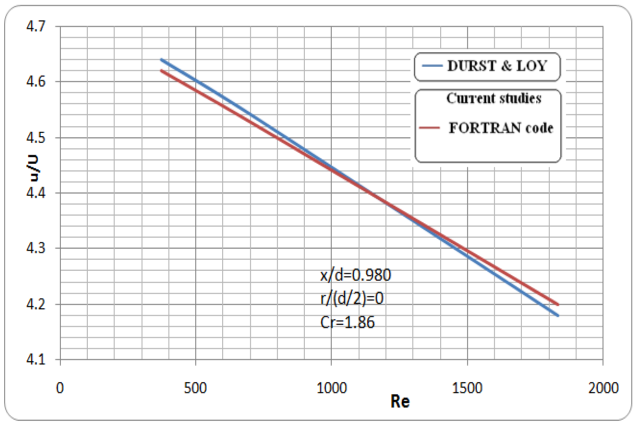


Figure (9) Comparison the axial velocity profile at (Z/d=0.980, r/ (d/2) =0) For the FORTRAN CODE, fluent software & Durst

## 4.2 Case 2 Non- axisymmetric pipe

The flow in a pipe with sudden contraction in laminar flow at different Reynolds numbers is studied at three ratios between the large and small diameters(10).

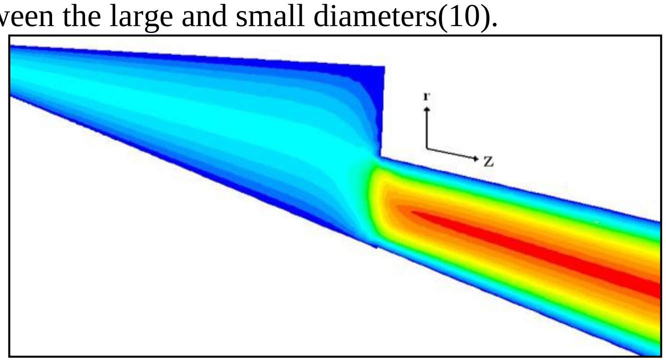


Figure (10) Non-axisymmetric pipe with contraction ratio = 2

**Case2.A:**

**Contraction ratio = 2**

The comparisons of numerical results using the Fortran code with Fluent in 3D are shown in Figure (11) and Figure (12) for two different Reynolds numbers and Cr=2. The results are in good agreement with Fluent results as illustrated in both Figures.

$Re_d = 500, 1000$

Cr=2

Comparing the axial velocity profile for the Fortran code and Fluent software.

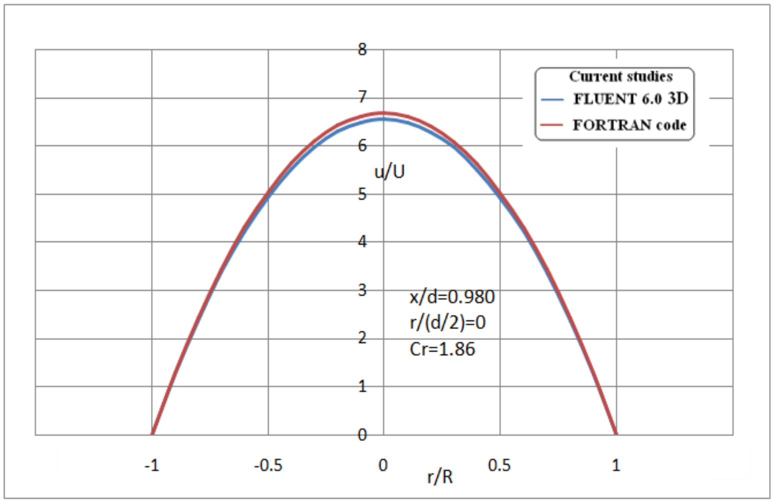


Figure (11) Comparing of the axial velocity profile for the FORTRAN code and fluent software

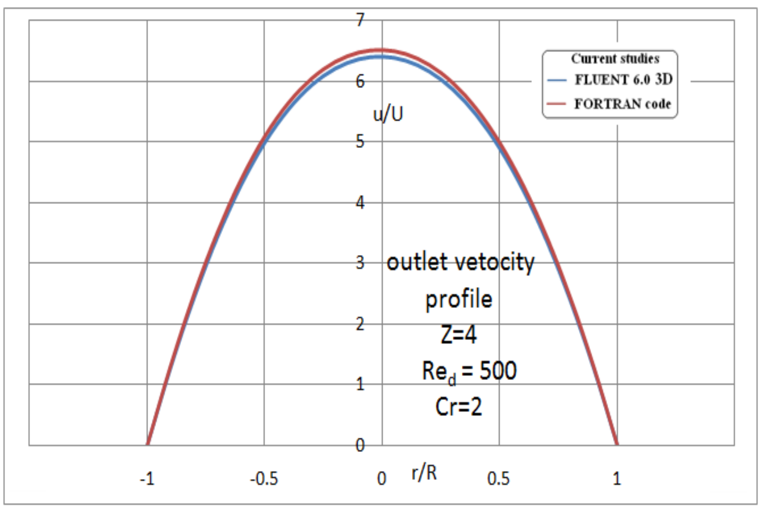


Figure (12) Comparing of the axial velocity profile for the FORTRAN code and fluent

Figures (13) and (15) show the evaluation of the velocity contours in the sudden contraction for various Reynolds numbers and Cr=2. From the previous Figures, it is easy to show that the fluid separated at the entrance region next to the upper wall of the small pipe, and a small separation region exists in the concave corner of the contraction. This separation increases with Reynolds number increase. A recirculation zone is detected in the entrance and concave corner. Dimensions of this recirculation flow region depend on the values Reynolds number [2].

The pressure drop encountered at the entrance of the small pipe, caused by the sudden contraction under the conditions of laminar flow, is of interest in the data analysis. It has a large gradient in the axial direction and a small gradient in the radial direction, as shown in Figures (14) and (17). The pressure coefficient distributions along the contraction of the pipe are presented in Figure (4.18). The pressure loss coefficient (Cp) is obtained from the ratio of static pressure to dynamic pressure, and the computed skin friction factor along the contraction of the pipe is shown in Figure (4.19).

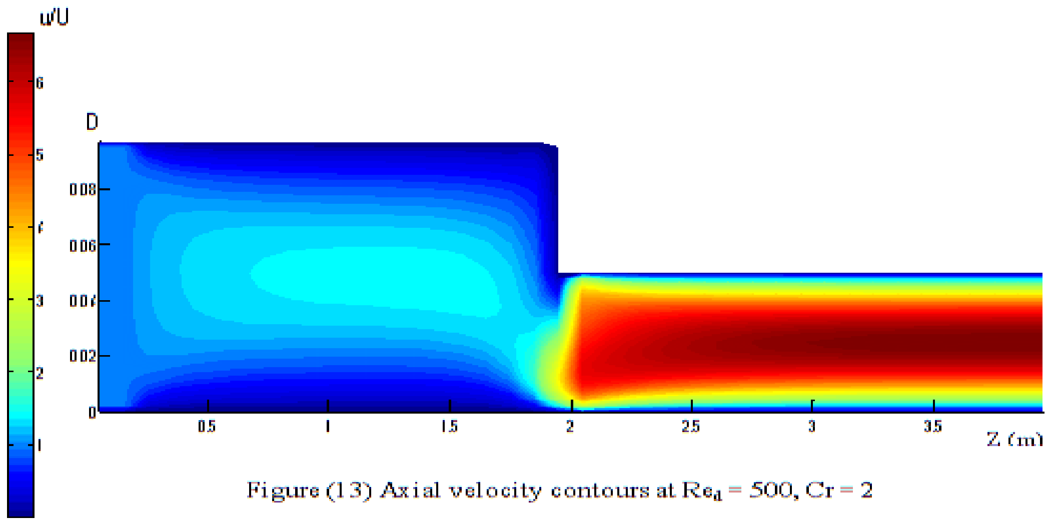


Figure (13) Axial velocity contours at $Re_d = 500$, Cr = 2

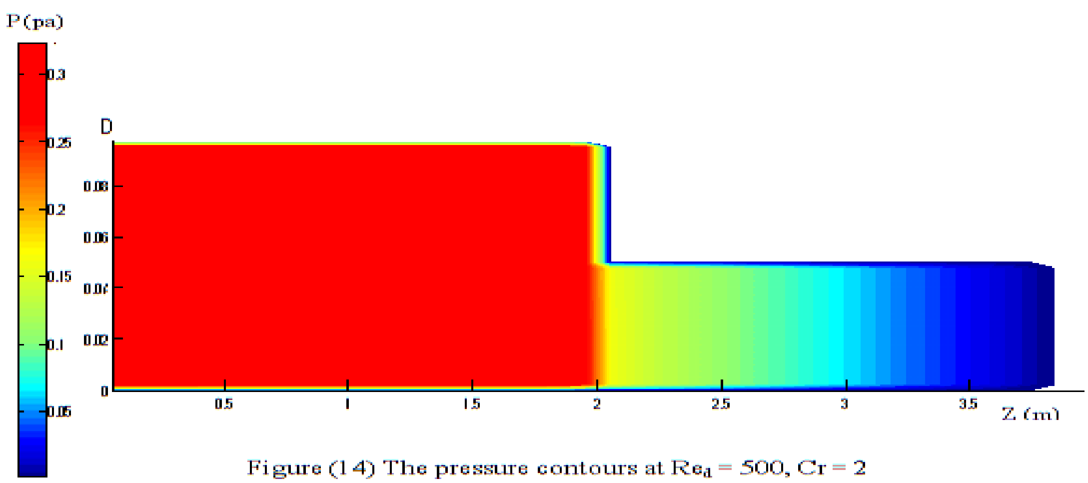


Figure (14) The pressure contours at $Re_d = 500$, Cr = 2

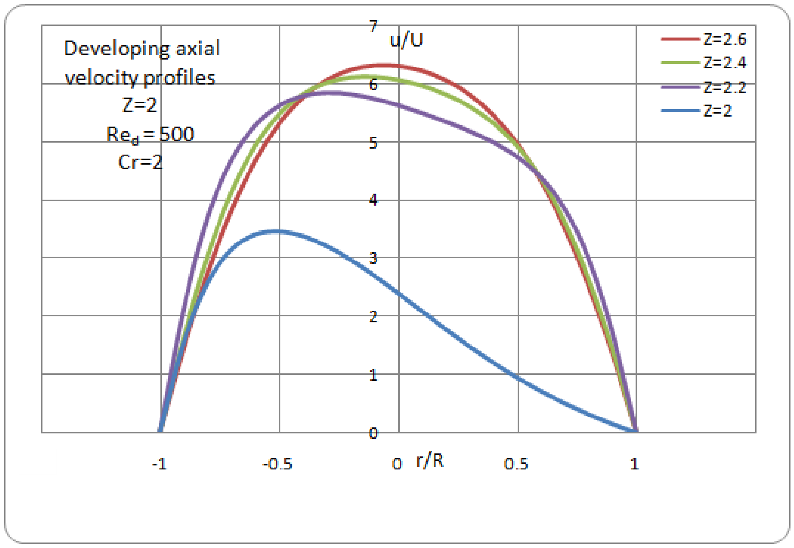


Figure (15) Developing axial velocity profiles for $Re_d = 500$, $Cr = 2$

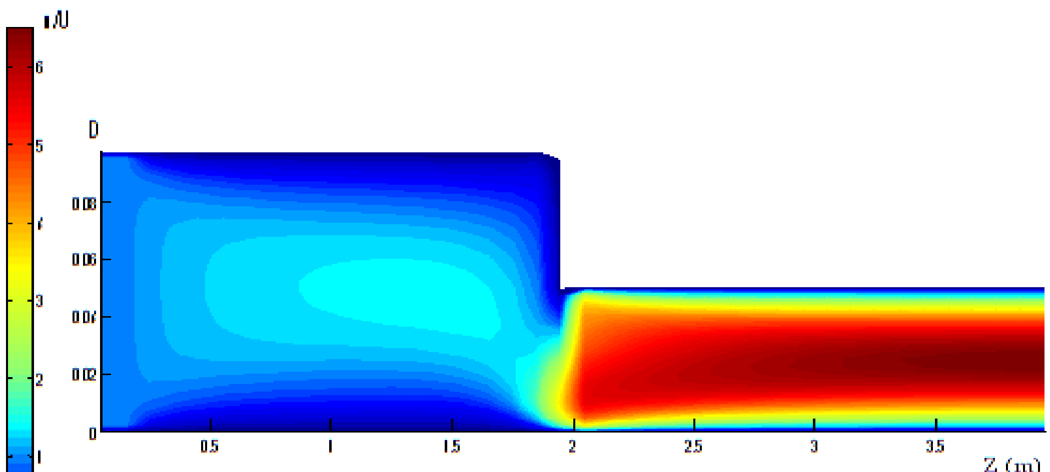


Figure (16) Axial velocity contours at $Re_d = 1000$, $Cr = 2$

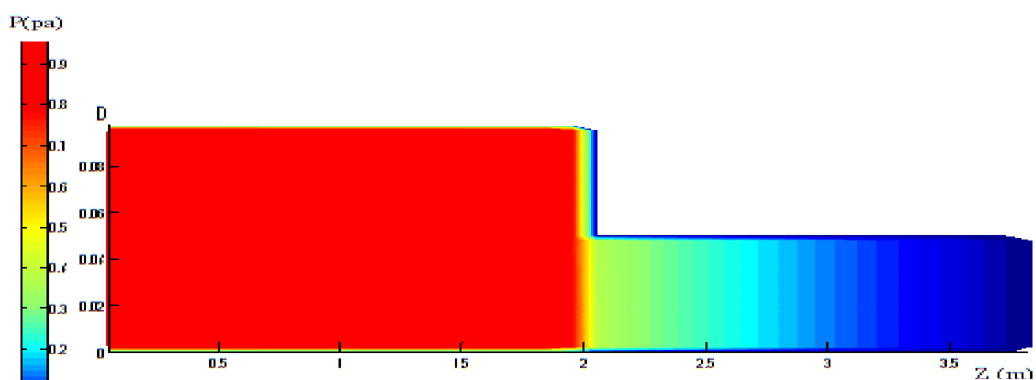


Figure (17) The pressure contours at $Re_d = 1000$, $Cr = 2$

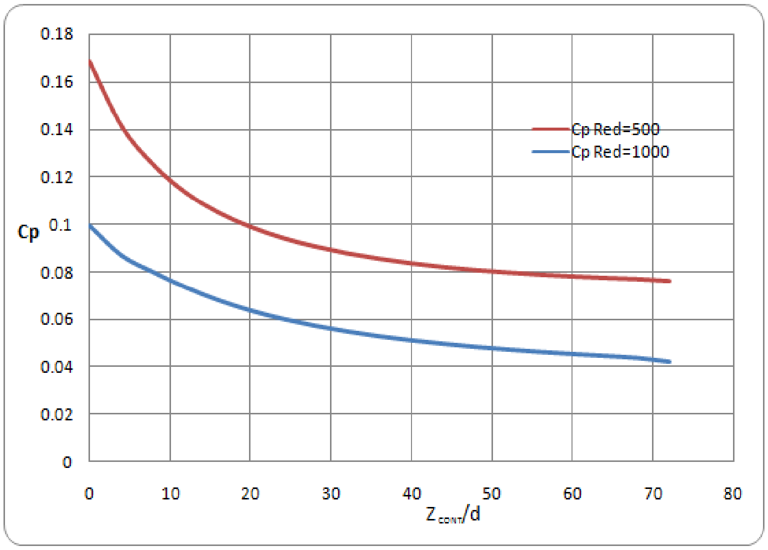


Figure (18) Local pressure coefficient for Cr = 2

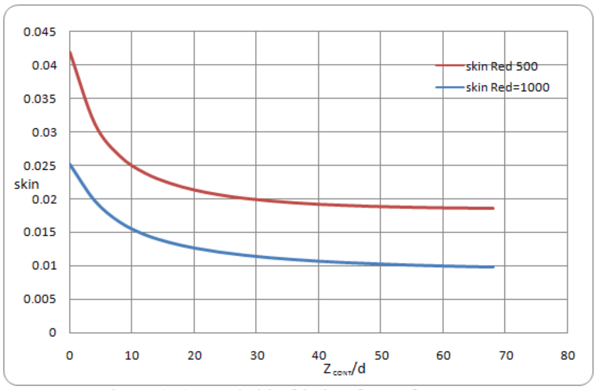


Figure (19) Local skin friction factor for Cr = 2

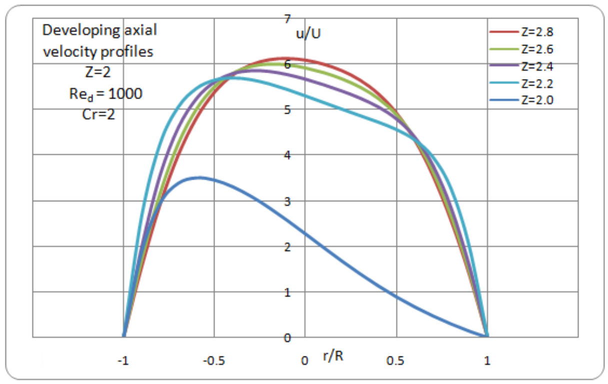


Figure (20) Developing axial velocity profiles for $Re_d = 1000$, Cr = 2

**Case2.B:**

**Contraction ratio =3**

The comparisons of numerical results using the Fortran cod with Fluent in 3D are shown in Figure (21) and Figure (22) for two different Reynolds numbers and Cr=3. The results are in good agreement with Fluent results, as illustrated in Figure (21) and Figure (22). As is evident from the Figures, good agreement between the Fluent and current results is obtained.

Red = 750, 1500

Cr =3

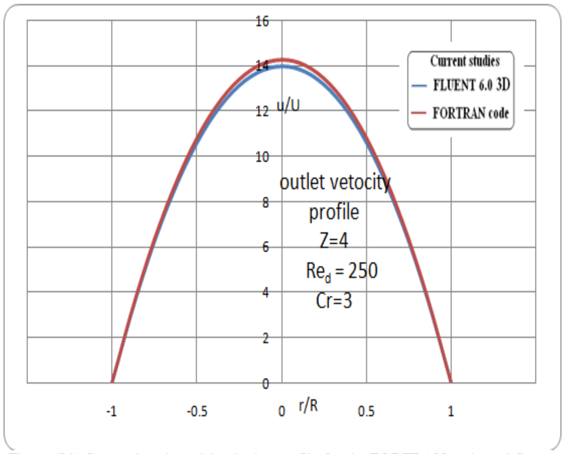


Figure (21) Comparing the axial velocity profile for the FORTRAN code and fluent software.

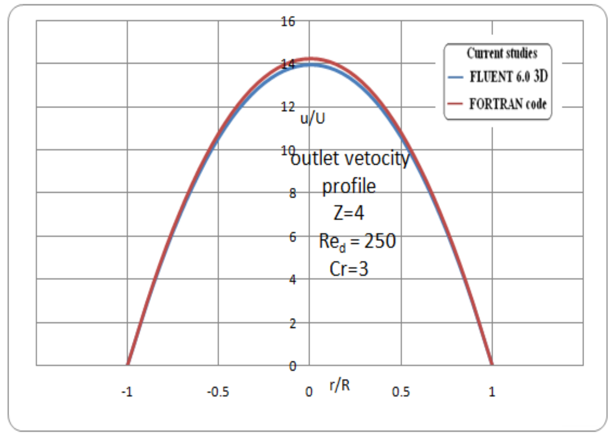


Figure (22) Comparing the axial velocity profile for the FORTRAN code and fluent software.

Figures (23) and (26) show the evaluation of the velocity contours in the sudden contraction for various Reynolds numbers and Cr=3. From the previous Figures, it is easy to show that the fluid-separated region is increasing at the entrance region next to the upper wall of the small pipe and increase a small separation flow existing in the concave corner of the contraction. This separation increases with the Reynolds number increase and the contraction ratio from case 1. The pressure drop increase with the Reynolds number increases. With the contraction ratio increase from case 1, it has a large gradient in the axial direction and a small gradient in the radial direction, as shown in Figures (24) and (27). The pressure coefficient (Cp) distribution along the contraction of the pipe is presented in Figure (28). Moreover, the skin friction factor along the contraction of the pipe is presented in Figure (29).

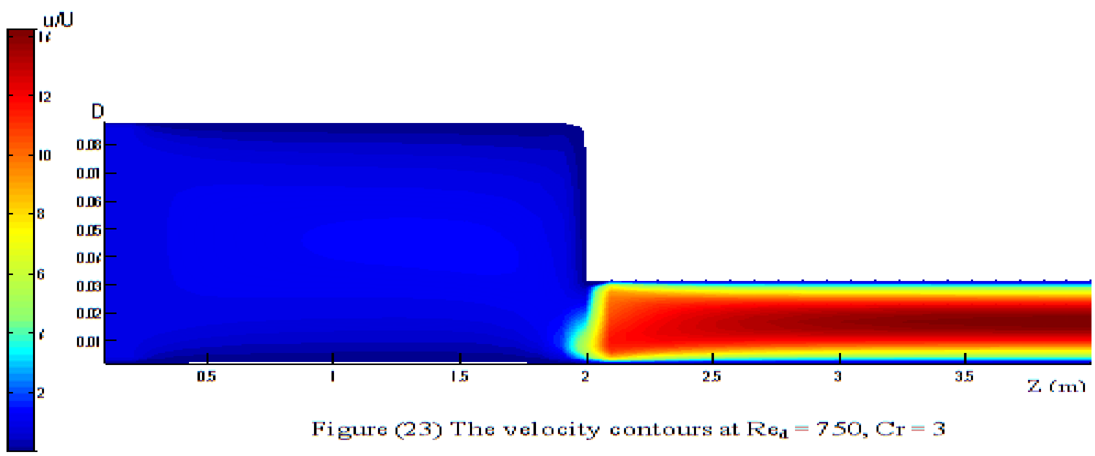


Figure (23) The velocity contours at $Re_d = 750$, $Cr = 3$

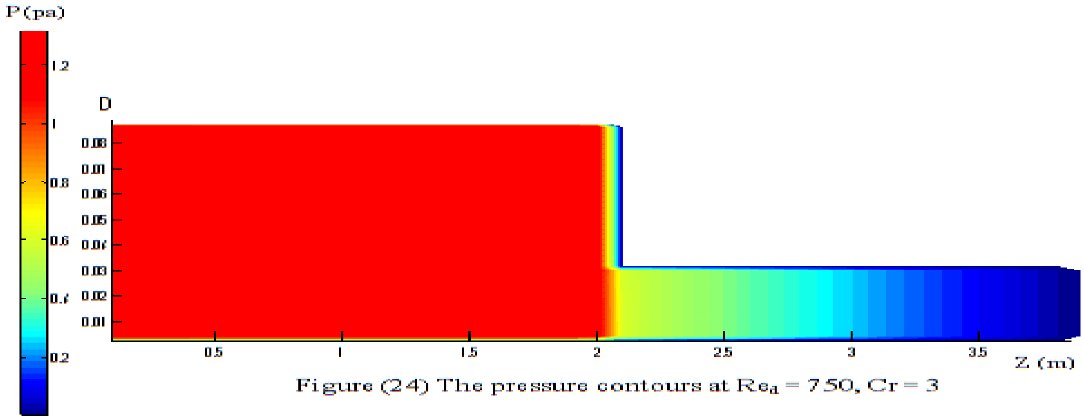


Figure (24) The pressure contours at $Re_d = 750$, $Cr = 3$

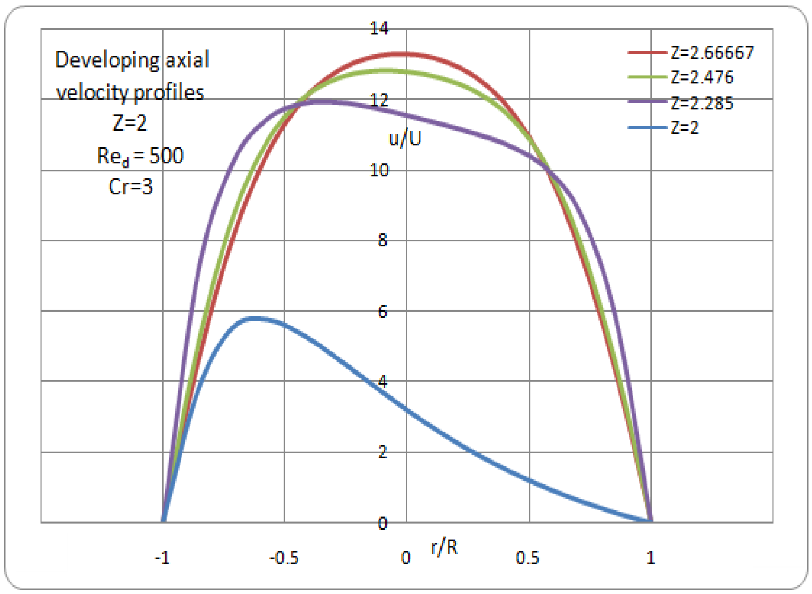


Figure (25) Developing axial velocity profiles for $Re_d = 750$, $Cr = 3$

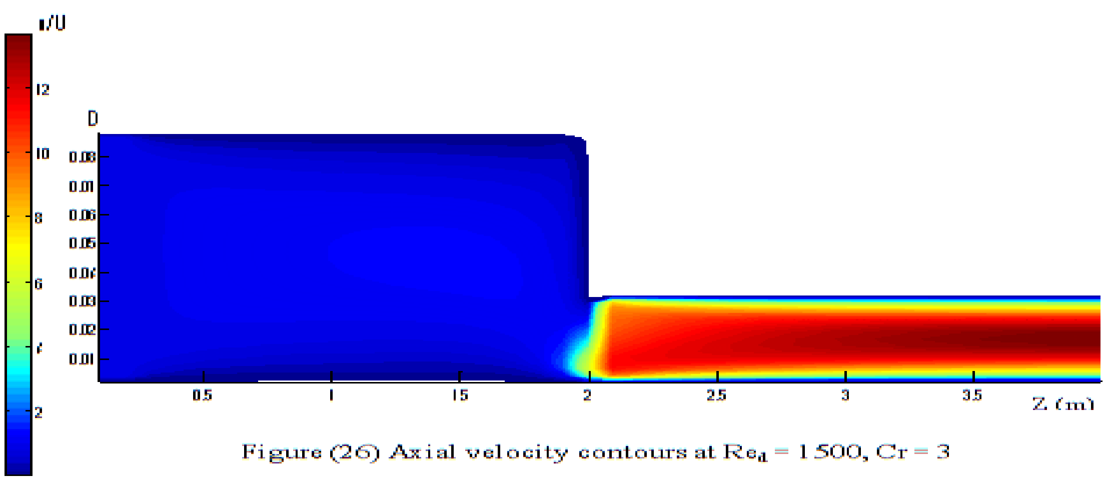


Figure (26) Axial velocity contours at $Re_d = 1500$, $Cr = 3$

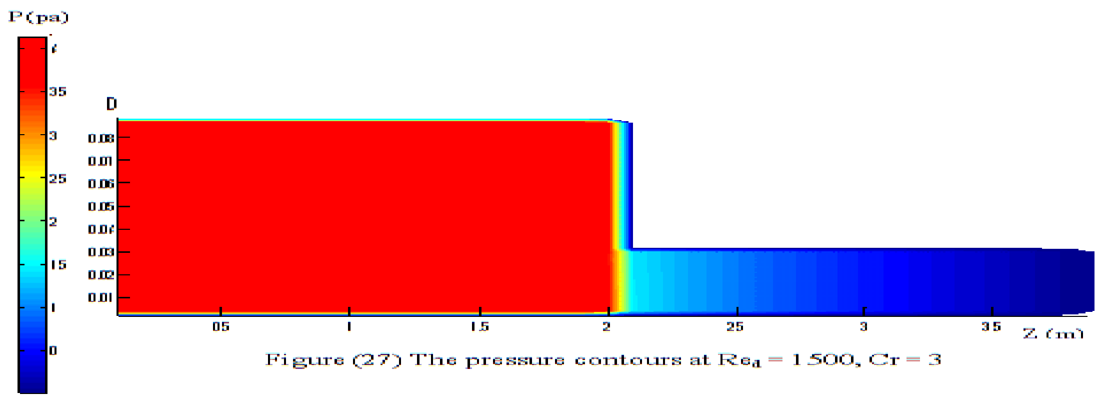


Figure (27) The pressure contours at $Re_d$ = 1500, Cr = 3

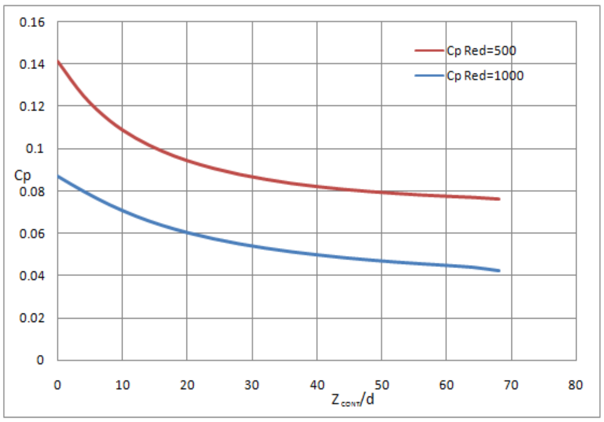


Figure (28) Local pressure coefficient for Cr = 3

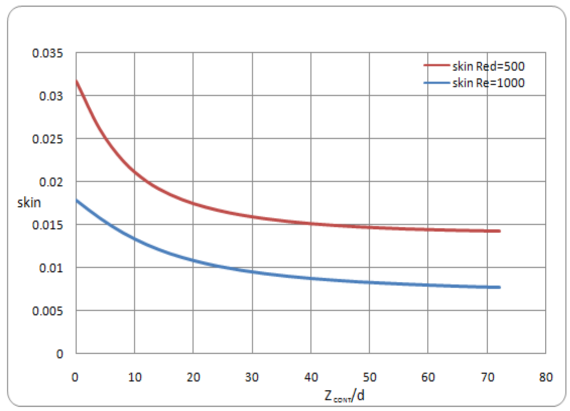


Figure (29) Local skin friction factor for Cr = 3

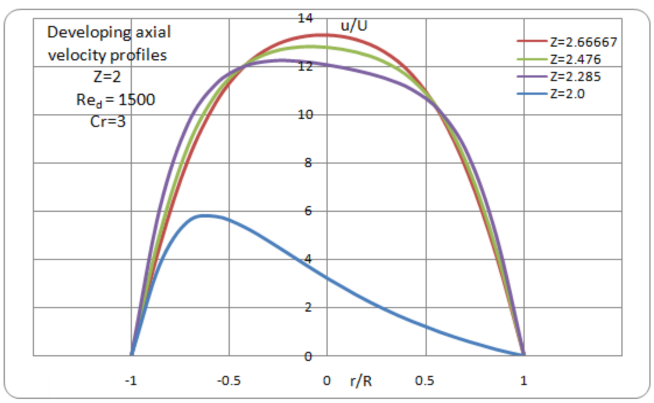


Figure (30) Developing axial velocity profiles for $Re_d$ = 1500, Cr = 3

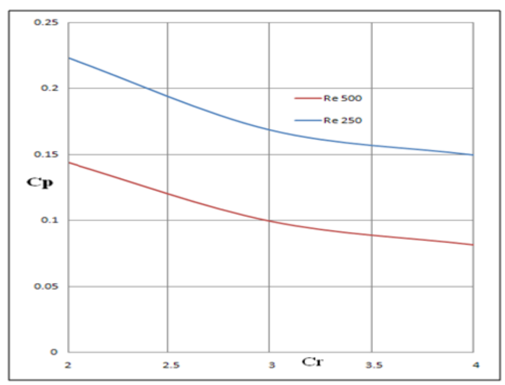


Figure (31) The pressure coefficient for flow through sudden area changes.

Figures (13), (16), (23) and (26) show the evaluation of the velocity contours in the sudden contraction for various Reynolds numbers and various contraction ratios. It is shown that the fluid separated at the entrance region is next to the upper wall of the small pipe, and another small separation flow region exists in the concave corner of the contraction. This separation increases with the Reynolds number increase as well as the increase of contraction ratio. A circulation zone is detected in the entrance and concave corner; this recirculation flow region's dimensions depend on

Reynolds number and contraction ratios**.** The length of the developing region is defined as the distance from the pipe inlet to the point where the centerline velocity achieves 95% of its developed value. The developing region is plotted in Figures (15), (20), (25 and (30) for various Reynolds numbers and different contraction ratios using the parabolic model**.**

Though the above results show a significant overshoot for fluid appearing at the entrance region, all results are similar for moderate Reynolds numbers**.**

Regarding the pressure losses caused by the sudden contraction, computations are performed for various Reynolds numbers and contraction ratios to yield the results given in Figures (14), (24) and (27). The computed pressure coefficient distributions along the contraction of the pipe are presented in Figures (4.18) and (28). The computed skin friction factor along the contraction of the pipe is provided in Figures (19) and (29).

## 5. Summary and Conclusions

The employment of the numerical prediction procedure is instrumental in obtaining reliable information on pipe flow with sudden changes in the cross-sectional area. The present research constitutes a comprehensive numerical study of laminar flow in a pipe with sudden contraction. Various parameters related to flow conditions and the geometry of interest are varied to demonstrate their influence on the velocity field and the pressure distribution; these parameters are the inlet flow velocity and the contraction ratio in the pipe diameter.

Numerical schemes are developed frequently to obtain better results. In this study, the flow behaviour in a pipe with sudden contraction is studied numerically using the finite volume method. A general code with automatic mesh generation is created and validated compared to numerical and experimental results; this code was carefully checked and optimized to yield reliable predictions for pipe flows with a sudden contraction in the cross-sectional area. Finally, computational results are presented and compared with the available experimental results.

One can outline here some conclusive remarks on a laminar flow in a pipe with sudden contraction present solution, as follows:

- ❖ The profiles and contours of the axial flow velocity show that a separated flow region exists in the contraction's concave corner.
- ❖ The separation increases with the Reynolds number as well as the increase of the contraction ratios.
- ❖ he pressure behaviour of the flow in the pipe with sudden contraction showed decreases in static pressure rapidly behind the sudden contraction to its minimum value at the contraction region.
- ❖ The pressure decreases again along the downstream due to the recovery as a result of velocity increases.

❖ The sudden contraction in the geometry causes pressure losses in the pipe flow.

| | |
|---|---|
| **Nomenclature** | $\bar{U}$ average velocity (m/s) |
| $A$ Cross-sectional area ($m^2$) | $u$ Velocity component in the x direction (m/s) |
| $C_f$ Local wall skin friction coefficient | |
| Cr contraction radio ($D_1/D_2$) | $U$ Non-dimensional flow speed |
| $C_v$ Specific heat at constant volume (kj/kg.k) | $v$ Velocity component normal to the wall (m/s) |
| $D$ pipe diameter (m) | $\bar{V}$ Velocity vector (m/s) |
| $d$ Diameter for small pipe(m) | $V_c$ The centerline velocity (m/s) |
| i Specific internal energy(kj/kg) | $Z$ Distance along the wall in the flow direction |
| $k$ Thermal conductivity (w/m.k) | |
| $L$ Length (m) | $Z_{CONT}$ The length of small pipe |
| $\dot{m}$ Mass flow rate (kg/s) | $z,r,\theta$ Cylindrical coordinates |
| $P$ Static pressure (kPa) | |
| $Q$ The flow rate ($m^3/s$) | **Greek symbols** |
| $q$ Heat flux ($W/m^2$) | $\delta$ Thickness (m) |
| $R$ Gas constant | $\phi$ Generic variable (scalar variable) |
| $Re$ Reynolds number ($\rho \bar{U} D/\mu$) | $\mu$ dynamic viscosity (kg/ms) |
| $Re_{max}$ Reynolds max ($\rho \bar{U}_{max} D/\mu$) | $\rho$ density ($kg/m^3$) |
| | $\tau$ Wall shear stress ($N/m^2$) |

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